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Article

Financing Decisions in a Green Closed-Loop Supply Chain under Demand Uncertainty and Government Consumer Subsidies

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Abstract

Amid growing environmental pressures and resource constraints, green closed-loop supply chains (GCLSCs) play a vital role in achieving sustainable economic development. This study examines a GCLSC composed of one manufacturer and one retailer under uncertain market demand and government consumer subsidies. Using a Stackelberg game framework, we analyze the manufacturer's production and financing decisions across three scenarios: no capital constraint, advance-payment financing, and equity financing. The results show that demand uncertainty, consumer price sensitivity, and green preference significantly affect optimal decisions. Under advance-payment financing, manufacturers raise wholesale prices but reduce product greenness and recycling levels, lowering their own profit while improving retailer gains. In contrast, equity financing aligns with the unconstrained benchmark, preserving environmental performance and overall efficiency. Numerical analyses confirm the theoretical findings. The study provides managerial insights for promoting green financing, enhancing consumer awareness, and fostering innovation in sustainable supply chains.

Keywords: Green closed-loop supply chain; Government consumer subsidy; Capital constraint; Financing mode; Demand uncertainty

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1. Introduction

Against the backdrop of escalating environmental degradation and resource scarcity,governments and industries worldwide are intensifying efforts to mitigate supply chain pollution and enhance resource efficiency across production and logistics processes. A prominent example is China's announcement at the 75th United Nations General Assembly of its "dual carbon" targets—peaking carbon emissions by 2030 and achieving carbon neutrality by 2060—which was further substantiated by a comprehensive policy document issued by the Central Committee and the State Council in September 2021. In this context of accelerating green transition, green closed-loop supply chains (GCLSCs), which integrate product recycling and remanufacturing into forward logistics, have attracted significant attention from both practitioners and scholars.

Existing research has explored multiple dimensions of GCLSCs. For instance, Zhang et al. (2021) demonstrate that retailer fairness concerns can improve green product quality, recycling rates, and consumer welfare. Zhang Meng (2022) investigates government subsidy mechanisms under different power structures and derives optimal strategies for supply chain actors and policymakers, underscoring the instrumental role of public policy in advancing green production and circularity. In practice, subsidies targeting either producers or consumers—such as those for energy-efficient appliances and new energy vehicles—have been widely implemented to stimulate green consumption. Consequently, a stream of literature has emerged examining how subsidy policies influence supply chain decisions. Sinayi and Barzoki (2018) adopt a game-theoretic approach to analyze the impact of subsidies on pricing and greenness, while Ma et al. (2018) compare pre- and post-subsidy decisions in a dual-channel closed-loop supply chain. Xing and Li (2019) and Meng et al. (2021) further evaluate the differential effects of consumer-oriented versus producer-oriented subsidies, highlighting the efficacy of demand-side incentives in promoting remanufactured products.

Another relevant research thread concerns supply chain financing under capital constraints. Several studies have introduced financial mechanisms into settings such as dual-channel or low-carbon supply chains. For example, Li et al. (2020) and Zhen et al. (2020) explore financing modes—including trade credit, bank loans, and platform financing—for capital-constrained manufacturers. Wu et al. (2022) and Yu et al. (2021) extend this inquiry to carbon-constrained environments. However, these studies largely rely on deterministic demand assumptions and do not incorporate government green subsidies. In reality, demand uncertainty is pervasive and plays a critical role in shaping operational and financial decisions. Although some scholars, such as Buzacott et al. (2004), and Guo and Xin (2022), have incorporated stochastic demand into financing models, their work does not fully address the intersection of financing mechanisms, consumer subsidies, and green closed-loop structures.

A systematic review of the literature reveals a notable gap: little research has simultaneously considered demand uncertainty, consumer-targeted subsidies, and financing strategies within GCLSCs. To address this gap, this paper investigates a GCLSC consisting of a single manufacturer and retailer under stochastic demand and government subsidies. We develop a Stackelberg game-theoretic model to analyze the manufacturer's optimal financing choice among three modes—equity financing, advance-payment financing, and bank loans (representing the unconstrained benchmark)—and examine their implications for production quantity, product greenness, recycling rate, and supply chain performance.

The remainder of this paper is organized as follows. Section 2 formulates the problem and outlines key assumptions, Section 3 presents the model and analyzes decisions under each financing mode, Section 4 compares equilibrium outcomes and presents key propositions,Section 5 concludes with managerial and policy implications.

2. Description and basic assumptions

This paper examines a green closed-loop supply chain comprising a single manufacturer and a single retailer. In the forward flow, the manufacturer is responsible for production and supplies products, obtained through two sourcing channels, to the downstream retailer at a unit wholesale price ,The retailer subsequently sells these products to consumers at a unit retail price p. In the reverse flow, the manufacturer collects used products from consumers at a unit recycling price c for remanufacturing.

In this paper, the manufacturer is considered the leader in a Stackelberg game. The decision-making sequence in the supply chain is as follows: First, the government provides a fixed subsidy to consumers, resulting in an actual purchase price of $z = p - k$, Next, as the leader of the game, the manufacturer determines the wholesale price , the green degree , and the collection rate of used products after becoming aware of the government subsidy policy. Then, the retailer, as the follower, decides the retail price and the order quantity after observing the manufacturer's decisions.

The market demand faced by the retailer is given by: $D = a - \theta_2 p + \mu g$, where a is a random variable representing the market demand or scale when the price $p = 0$ and the government subsidy $k = 0$ To simplify the model, it is assumed that a follows a uniform distribution on the interval $[a,d]$. Thus, the actual demand D follows a uniform distribution on the interval:

$[a-\theta_2 p+\mu g, d-\theta_2 p+\mu g]$, Both the manufacturer and the retailer are assumed to be risk-neutral and possess symmetric information. The parameters satisfy the following condition: $-2Z(a-d)-p\mu^2>0$ Detailed descriptions of the notations are provided in Table 1.

Table 1. Parameter Symbol and Parameter Definition

Parameter Symbol	Parameter Definition
p	Retail price of the product
w	Wholesale price of the product
g	Green degree of the product
λ	Investment rate in green manufacturing cost
D	Market demand
B	Manufacturer's initial capital
C_1	Unit production cost of new products
C_2	Unit production cost of remanufactured products
Q	Retailer's order quantity
θ_1	Conversion rate of used products into new products
A	Average recycling price paid by the manufacturer to consumers for collected products
μ	Sensitivity coefficient of the green degree
k	Subsidy amount provided by the government to consumers for product purchase
θ_2	Price sensitivity coefficient
δ	Dividend ratio of risk investment
γ	Advance payment discount rate
τ	Collection rate of used products
f	Amount of equity financing
z	Actual purchase price per unit product for consumers
$C_L(C_L>0)$	Recycling scale parameter

Based on the Stackelberg game framework and the context of the green closed-loop supply chain, the following assumptions are made in this study:

(1) It is assumed that the manufacturer and the retailer have symmetric information and are both risk-neutral.

(2) The manufacturer produces both new products made from raw materials and remanufactured products made from collected used products. The two types of products are assumed to be indistinguishable to consumers and are sold at the same market price.

(3) To ensure the rationality and feasibility of recycling and remanufacturing, the cost saving from remanufacturing is positive, $C_1-C_2>0$. The unit transfer price paid by the manufacturer to the retailer is greater than the unit price paid to consumers for recycling used products, $w>A$,To maintain the effectiveness of the closed-loop supply chain, $C_1-C_2>w>A$.

(4) The unit cost of remanufacturing using used products $C2$ is lower than the unit cost of producing with new materials C1, so the unit cost saving is $C1-C2$. To mitigate adverse effects from quality variations in used products, $C2$ is defined as the average unit production cost. To incentivize the manufacturer to engage in recycling, $A<C1-C2$ must hold.

(5) The manufacturer bears the additional cost of green technology innovation. the R&D cost for producing green products is assumed to be λg^2 , where λ represents the efficiency of the manufacturer's fixed R&D investment.

(6) The market demand D exceeds the quantity of remanufactured products C_L.

(7) The government implements a clear subsidy policy, providing a fixed subsidy to consumers who purchase green products.

(8) It is assumed that the retailer has sufficient capital to finance the manufacturer and does not face bankruptcy risk itself.

3. Model building and analysis

3.1. No funding constraints

Under the consumer subsidy decision model in a manufacturer-dominated market, the manufacturer's primary revenue stream primarily comes from wholesaling products—jointly produced using both raw materials and recycled used products—to the retailer. The manufacturer's main costs include the production cost of manufacturing new products, the expense associated with collecting used products from consumers, and the R&D cost for developing green products. Accordingly, the profit function of the manufacturer is given as follows:

$$\Pi_M = [w - C_1 + (C_1\theta_1 - A - C_2)\tau]Q - \lambda g^2 - C_L\tau^2$$

The retailer's revenue is primarily generated from sales to consumers, while its main expenditure consists of the wholesale payments made to the manufacturer. Accordingly,the retailer's profit function is formulated as follows:

$$\begin{aligned}\Pi_R &= pE(\min(D,Q)) - wQ \\ &= \frac{p}{d-a}[Q(d-\theta_2 z+\mu g) - \frac{Q^2+(a-\theta_2 z+\mu g)^2}{2}] - wQ\end{aligned}$$

When the manufacturer faces no financial constraints, the expected profit functions of the retailer and the manufacturer are given by:

$$\Pi_M = [w - C_1 + (C_1\theta_1 - A - C_2)\tau]Q - \lambda g^2 - C_L\tau^2$$

$$\begin{aligned}\Pi_R &= pE(\min(D,Q)) - wQ \\ &= \frac{p}{d-a}[Q(d-\theta_2 z+\mu g) - \frac{Q^2+(a-\theta_2 z+\mu g)^2}{2}] - wQ\end{aligned}$$

The optimal order quantity, wholesale price, green degree, and collection rate of used products, derived by using backward induction, are as follows:

$$Q^* = \frac{2\lambda C_L(pXY + C_1X^2)}{\mu^2 p^2 C_L + 4\lambda pC_L X + \lambda X^2 N^2}$$

$$w^* = \frac{\lambda p(-2pC_L Y + 2C_1C_L X - XYN^2) + p^2\mu^2 C_1 C_L}{\lambda X^2 N^2 + \mu^2 p^2 C_L + 4\lambda pC_L X}$$

$$g^* = \frac{-p\mu(pY + XC_1)C_L}{\lambda X^2 N^2 + \mu^2 p^2 C_L + 4\lambda pC_L X}$$

$$\tau^* = \frac{\lambda N(XYp + C_1X^2)}{\lambda X^2 N^2 + \mu^2 p^2 C_L + 4\lambda pC_L X}$$

Among, $X = a - d, Y = d - \theta_2 z, N = C_1\theta_1 - A - C_2$

In green supply chains, as consumer preference for green products increases, manufacturers need to allocate more capital toward enhancing product greenness. Assuming the capital market is perfectly competitive with a risk-free interest rate, the manufacturer's initial capital is denoted as $B(B > 0)$, and the initial capital required for optimal production quantity and optimal greenness R&D under no financial constraints is:

$$B_N = [C_1(1 - \tau\theta_1)D + C_2\tau D] + \lambda g^2 + D\tau A + C_L\tau^2 \quad .$$

If the initial capital $B < B_N$ the manufacturer faces capital constraints. This paper adopts advance payment financing and equity financing models to address the manufacturer's financial constraints.

3.2. Advance Payment Financing

Under the advance payment financing model, to incentivize the retailer to participate in financing, the manufacturer attracts the retailer by offering an advance payment discount. The advance payment discount rate, denoted as $\gamma(\gamma \geq 0)$, is assumed to be externally given. When the manufacturer has insufficient initial capital, the well-funded retailer, to ensure product availability and without compromising its own interests, has an incentive to provide advance payment financing to the manufacturer.

Let w_1, Q_1, g_1, τ_1 denote the wholesale price, retailer's order quantity, product greenness, and collection rate of used products, respectively, under the advance payment financing model. The expected profits of the retailer and the manufacturer are given as follows:

$$\Pi_{R1} = pE(\min(D, Q_1)) - w_1Q_1 + \gamma[B_N - B]$$

$$\Pi_{M1} = [w_1 - C_1 + (C_1\theta_1 - A - C_2)\tau]Q_1 - \lambda_1 g_1^2 - C_L\tau_1^2 - \gamma[B_N - B]$$

Using backward induction, the above equations are solved. Therefore, the optimal decisions for the manufacturer and retailer are derived as follows:

$$Q_1^* = \frac{2\lambda pC_L XY(\gamma+1) + \lambda C_1C_L(\gamma+1)(2\gamma+1)X^2 + \mu^2 p\gamma C_1C_L X - \lambda\gamma X^2 YN^2}{4\lambda pC_L X(\gamma+1) + \mu^2 p^2 C_L + \lambda X^2 N^2}$$

$$w_1^* = \lambda pz\theta_2(\gamma+1)(2pC_L + XN^2)/[\mu^2 p^2 C_L + 4\lambda pC_L(\gamma+1)X + \lambda X^2 N^2 - 2\lambda dp^2 C_L(\gamma+1)$$
$$+\mu^2 p^2 C_1 C_L(\gamma+1) + \lambda dXN^2(\gamma+1) - 2\lambda C_1 C_L(\gamma+1)(2\gamma+1)X]$$
$$[\mu^2 p^2 C_L + 4\lambda pC_L(\gamma+1)X + \lambda X^2 N^2]$$

$$g_1^* = \frac{-\mu pC_L(pY + C_1 X)}{\mu^2 p^2 C_L + 4\lambda pC_L(\gamma+1)X + \lambda X^2 N^2}$$

$$\tau_1^* = \frac{\lambda(pY + C_1 X)XN}{\mu^2 p^2 C_L + 4\lambda pC_L(\gamma+1)X + \lambda X^2 N^2}$$

By comparing the optimal decisions under advance payment financing with those in the scenario without financial constraints, it can be observed that, relative to when compared with the optimal decisions without financial constraints, the manufacturer provides an advance payment discount to the retailer and correspondingly increases the wholesale price to ensure the maximization of its own interests.

3.2. Equity Financing

Under the equity financing model, venture capitalists typically do not require the manufacturer to repay principal and interest at maturity, but instead adopt a dividend distribution approach at the end of the period. In the initial financing stage, the manufacturer uses the initial capital required for optimal production volume and optimal greenness R&D under no financial constraints as a baseline. The dividend ratio for venture capital is set as δ . In case of insufficient capital, $B_N - B = f, B < B_N$, meaning the manufacturer has no excess idle funds and utilizes the entire financing amount exactly.

Let w_3, Q_3, g_3, τ_3 denote the wholesale price, retailer's order quantity, product greenness, and collection rate of used products, respectively, under the equity financing model. The expected profits of the retailer and the manufacturer are given as follows:

$$\Pi_{R3} = pE(\min(D, Q_3)) - w_3 Q_3$$

$$\Pi_{M3} = [1-\delta]\left\{[w_3 - C_1 + (C_1\theta_1 - A - C_2)\tau_3]Q_3 - \lambda_3 g_3^2 - C_L \tau_3^2 + B + f - B_N\right\} - B$$

Using backward induction to solve the model, the optimal decisions for the manufacturer and retailer when the venture capital dividend ratio is set as δ are derived as follows:

$$Q_3^* = \frac{4\lambda pC_L XY + 8\lambda C_1 C_L X^2}{8\lambda pC_L X + 4\lambda X^2 N^2 + \mu^2 p^2 C_L}$$

$$w_3^* = \frac{4\lambda p^2 Y + 2\mu^2 p^2 C_1 + 8\lambda pC_1 C_L X - 4\lambda pXYN^2}{\mu^2 p^2 C_L + 8\lambda pC_L X + 4\lambda X^2 N^2}$$

$$g_3^* = \frac{-\mu pC_L(pY + 2C_1 X)}{\mu^2 p^2 C_L + 8\lambda pC_L X + 4\lambda X^2 N^2}$$

$$T_3^* = \frac{2\lambda pXYN + 4\lambda C_1 N}{\mu^2 p^2 C_L + 8\lambda pC_L X + 4\lambda X^2 N^2}$$

Although this paper assumes that the retailer possesses sufficient capital, in reality, its financing capacity remains subject to upper limits. Should the manufacturer's required financing amount exceed the retailer's acceptable threshold (e.g., constrained by its cash flow or credit limits), advance payment financing may fail to fully meet the manufacturer's funding needs. In such cases, the manufacturer might need to seek partial bank credit or hybrid financing solutions. While this scenario falls beyond the scope of the current model, it warrants further exploration in future research, particularly regarding the interaction between retailer capital constraints and manufacturer financing strategies.

A comparison between the optimal decisions under equity financing and those under no financial constraints reveals that the optimal decisions under the equity financing model are independent of the venture capital dividend ratio. This is because, under direct investment from venture capitalists, the manufacturer is not required to repay principal and interest at maturity but instead distributes dividends after product delivery. In this scenario, both the venture capitalist and the manufacturer jointly own the enterprise and strive to maximize the firm’s profits. As a result, the funds obtained by the manufacturer from the venture capitalist are effectively equivalent to using internal capital. That is, under direct venture capital investment, the manufacturer’s optimal production level aligns with its optimal operational decisions in the absence of financial constraints, regardless of the venture capital dividend ratio.

4. Comparison and Analysis

Proposition 1: $\frac{\partial Q^*}{\partial \theta_1} < 0$ 、 $\frac{\partial Q^*}{\partial \theta_2} < 0$ 、 $\frac{\partial Q^*}{\partial \mu} < 0$ 、 $\frac{\partial Q^*}{\partial k} > 0$

Proof:

$$\frac{\partial Q^*}{\partial \theta_1} = -\frac{4\lambda C_L (X^2 C_1 + XYp) X^2 (C_1\theta_1 - A - C_2) C_1}{[4\lambda pC_L X + \lambda X^2 (C_1\theta_1 - A - C_2)^2 + \mu^2 p^2 C_L]^2} < 0,$$

$$\frac{\partial Q^*}{\partial \theta_2} = -\frac{2\lambda pzC_1 X}{4\lambda pC_L X + \lambda X^2 N^2 + \mu^2 p^2 C_L} < 0,$$

$$\frac{\partial Q^*}{\partial \mu} = -\frac{4\lambda C_L (X^2 C_1 + XYp) \mu p^2 C_L}{(4\lambda pC_L X + \lambda X^2 N^2 + \mu^2 p^2 C_L)^2} < 0,$$

$$\frac{\partial Q^*}{\partial k} = \frac{2\lambda C_L X \theta_2 p}{\lambda X^2 N^2 + \mu^2 p^2 C_L + 4\lambda pC_L X} > 0$$

Proposition 1 indicates that in the absence of financial constraints, Q^* decreases as μ、θ_1、θ_2 increases, while it increases with a rise in k . Specifically, when the manufacturer solely bears the costs of recycling and production for green products, the price sensitivity coefficient becomes a critical factor determining the retailer's order quantity. A higher price sensitivity coefficient leads the manufacturer to set a higher wholesale price, thereby reducing the retailer's order quantity. Conversely, a higher conversion rate of used products into new products enhances the manufacturer's cost savings, resulting in a lower wholesale price and a larger retailer order quantity. Moreover, an increase in the sensitivity coefficient toward greenness raises the required greenness level, elevating the manufacturer's costs and wholesale price, which in turn reduces the retailer's order quantity. Finally, a higher government subsidy on consumer purchases boosts consumer demand and the acceptable retail price, thereby increasing the retailer's order quantity.

Proposition 2: $\frac{\partial Q_1^*}{\partial \mu} > 0$ 、 $\frac{\partial Q_1^*}{\partial \theta_1} < 0$ 、 $\frac{\partial Q_1^*}{\partial \theta_2} > 0$ 、 $\frac{\partial Q_1^*}{\partial k} > 0$ 、 $\frac{\partial \tau_1}{\partial k} > 0$ 、 $\frac{\partial g_1}{\partial k} < 0$ 、 $\frac{\partial g_1}{\partial \gamma} > 0$ 、 $\frac{\partial \tau_1}{\partial \gamma} < 0$ 、 $\frac{\partial Q_1^*}{\partial \gamma} < 0$

Proof:

$$\frac{\partial Q_1^*}{\partial \mu} = \frac{\begin{array}{c}2\mu p C_L \lambda X(X^2N^2\gamma C_1 + XYN^2\gamma p + 4\gamma(\gamma+1)pC_1C_L \\ -2Y(\gamma+1)p^2C_L - C_L(\gamma+1)(2\gamma+1)XpC_1)\end{array}}{(4\lambda pC_L X(\gamma+1) + \mu^2p^2C_L + \lambda X^2N^2)^2} > 0,$$

$$\frac{\partial Q_1^*}{\partial \theta_1} = -\frac{\begin{array}{c}2\lambda X^2NC_1(\mu^2p\gamma C_1C_LX + \gamma Y\mu^2p^2C_L + 4\gamma X(\gamma+1)p\lambda YC_L \\ +2\lambda pC_LXY(\gamma+1) + \lambda C_1C_L(\gamma+1)(2\gamma+1)X^2\end{array}}{(4\lambda pC_LX(\gamma+1) + \mu^2p^2C_L + \lambda X^2N^2)^2} < 0,$$

$$\frac{\partial Q_1^*}{\partial \theta_2} = \frac{z\lambda\gamma X^2N^2}{4\lambda pC_LX(\gamma+1) + \mu^2p^2C_L + \lambda X^2N^2} > 0,$$

$$\frac{\partial Q_1^*}{\partial k} = \frac{\lambda\gamma X^2N^2\theta_2}{4\lambda pC_LX(\gamma+1) + \mu^2p^2C_L + \lambda X^2N^2} > 0,$$

$$\frac{\partial \tau_1}{\partial k} = \frac{p\lambda XN\theta_2}{4\lambda pC_LX(\gamma+1) + \mu^2p^2C_L + \lambda X^2N^2} > 0,$$

$$\frac{\partial g_1}{\partial k} = \frac{\mu p^2C_L\theta_2}{4\lambda pC_LX(\gamma+1) + \mu^2p^2C_L + \lambda X^2N^2} < 0,$$

$$\frac{\partial g_1}{\partial \gamma} = \frac{4\lambda p^2C_L^2X\mu(kp\theta_2 - p^2\theta_2 + XC_1 + dp)}{(4\lambda pC_LX(\gamma+1) + \mu^2p^2C_L + \lambda X^2N^2)^2} > 0,$$

$$\frac{\partial \tau_1}{\partial \gamma} = -\frac{4\lambda^2X^2NpC_L(kp\theta_2 - p^2\theta_2 + XC_1 + dp)}{(4\lambda pC_LX(\gamma+1) + \mu^2p^2C_L + \lambda X^2N^2)^2} < 0,$$

$$\frac{\partial Q_1^*}{\partial \gamma} = \frac{}{(4\lambda pC_LX(\gamma+1) + \mu^2p^2C_L + \lambda X^2N^2)^2} < 0$$

Proposition 2 indicates that when the manufacturer opts for the advance payment financing model under capital constraints, Q_1^* decreases with an increase in θ_1, but increases with a rise in θ_2、 μ . Under the advance payment financing model, the manufacturer offers an advance payment discount to the retailer. The price sensitivity coefficient determines the retailer's order quantity: a higher price sensitivity coefficient enhances the manufacturer's financing capacity and increases the financing amount, leading to a lower wholesale price and a higher retailer order quantity. When the manufacturer lacks sufficient funds to cover the costs of recycling and production for green products, its production remains constrained by the greenness sensitivity coefficient. A higher greenness sensitivity coefficient makes the product more attractive to the market, thereby increasing the retailer's order quantity. Additionally, a higher conversion rate of used products into new products reduces the manufacturer's costs, resulting in fewer discounts offered to the retailer, a relatively higher wholesale price, and a lower advance order quantity from the retailer.

Proposition 3: $\frac{\partial Q_3^*}{\partial \mu} < 0$、$\frac{\partial Q_3^*}{\partial \theta_1} < 0$、$\frac{\partial Q_3^*}{\partial \theta_2} < 0$、$\frac{\partial Q_3^*}{\partial k} > 0$、$\frac{\partial \tau_3}{\partial k} > 0$、$\frac{\partial g_3}{\partial k} > 0$.

proof:

$$\frac{\partial Q_3^*}{\partial \mu} = -\frac{2(4\lambda pC_L XY + 8\lambda C_1 C_L X^2)\mu p^2 C_L}{(8\lambda pC_L X + 4\lambda X^2 N^2 + \mu^2 p^2 C_L)^2} < 0,$$

$$\frac{\partial Q_3^*}{\partial \theta_1} = -\frac{8(4\lambda pC_L XY + 8\lambda C_1 C_L X^2)\lambda X^2 N C_1}{(8\lambda pC_L X + 4\lambda X^2 N^2 + \mu^2 p^2 C_L)^2} < 0,$$

$$\frac{\partial Q_3^*}{\partial \theta_2} = -\frac{4\lambda pC_L X z}{8\lambda pC_L X + 4\lambda X^2 N^2 + \mu^2 p^2 C_L} < 0,$$

$$\frac{\partial Q_3^*}{\partial k} = \frac{4\lambda pC_L X \theta_2}{8\lambda pC_L X + 4\lambda X^2 N^2 + \mu^2 p^2 C_L} > 0,$$

$$\frac{\partial \tau_3}{\partial k} = \frac{2\lambda pNX\theta_2}{8\lambda pC_L X + 4\lambda X^2 N^2 + \mu^2 p^2 C_L} > 0,$$

$$\frac{\partial g_3}{\partial k} = -\frac{\mu p^2 C_L \theta_2}{8\lambda pC_L X + 4\lambda X^2 N^2 + \mu^2 p^2 C_L} > 0$$

Proposition 3 indicates that when the manufacturer opts for the equity financing model under capital constraints, Q_3^* decreases with an increase in μ、θ_1、θ_2. Under the equity financing strategy, the manufacturer is not required to repay the principal and interest at maturity but must distribute dividends after product delivery. A higher greenness sensitivity coefficient leads to improved product quality and a correspondingly higher wholesale price, resulting in a reduced retailer order quantity. Similarly, a higher conversion rate of used products into new products raises the relative wholesale price, thereby increasing the manufacturer's profit but reducing the retailer's order quantity. Furthermore, a higher price sensitivity coefficient causes the retailer's profit to increase at a lower rate compared to the manufacturer's, which also contributes to a decrease in the retailer's order quantity.

Proposition 4: A comparison of the product greenness and the collection rate of used products under the advance payment financing strategy with those in the unconstrained scenario is as follows: $g^* > g_1^*, \tau^* > \tau_1^*$

Proof: Based on the equilibrium results, the comparisons are derived through mathematical operations between the equilibrium outcomes of the unconstrained model and the advance payment financing model. The calculation process is as follows:

$$g^* = \frac{-p\mu(pY + XC_1)C_L}{\lambda X^2 N^2 + \mu^2 p^2 C_L + 4\lambda pC_L X}$$

$$g_1^* = \frac{-\mu pC_L(pY + C_1 X)}{\mu^2 p^2 C_L + 4\lambda pC_L(\gamma + 1)X + \lambda X^2 N^2}$$

$$\tau^* = \frac{\lambda N(XYp + C_1 X^2)}{\lambda X^2 N^2 + \mu^2 p^2 C_L + 4\lambda p C_L X}$$

$$\tau_1^* = \frac{\lambda (pY + C_1 X) XN}{\mu^2 p^2 C_L + 4\lambda p C_L (\gamma + 1) X + \lambda X^2 N^2}$$

Proposition 4 indicates that when the manufacturer is responsible for recycling and acts as the leader in the supply chain, opting for the advance payment financing model under capital constraints leads to a reduction in the manufacturer's profit due to the advance payment discount offered to the retailer. To mitigate this, the manufacturer sets a higher wholesale price compared to the unconstrained scenario. Consequently, the product greenness and the collection rate of used products under the advance payment financing strategy are both lower than those in the unconstrained case. This occurs because the retailer, benefiting from the payment discount, faces lower ordering costs and thus sets a lower retail price. The reduced retail price attracts more consumers, leading to higher market demand compared to the unconstrained scenario. However, the increase in order quantity reduces the manufacturer's profit, prompting the manufacturer to lower both the product greenness and the collection rate of used products relative to the unconstrained case. As a result, the manufacturer's profit under the advance payment financing strategy is lower than in the unconstrained scenario, whereas the retailer's profit is higher.

5. Conclusions and Implications

This research investigates a manufacturer-led green closed-loop supply chain operating under stochastic demand and government subsidies, employing a Stackelberg game framework. Our findings indicate that consumer price sensitivity adversely impacts order quantities, whereas environmental preferences and subsidy policies positively influence market demand and retailer profitability. Under capital constraints, manufacturers utilizing advance-payment financing tend to increase wholesale prices while reducing product greenness and recycling rates to compensate for profit margins diminished by early payment discounts. In contrast, equity financing maintains the manufacturer's profit-maximization incentives, ensuring decisions regarding environmental attributes and recycling efforts remain consistent with unconstrained scenarios. Notably, while our model assumes uniform pricing for new and remanufactured products—reflecting prevailing market practices for standardized goods—future research could eplore differentiated pricing strategies for remanufactured items. Such approaches may stimulate price-sensitive demand segments and strengthen recycling incentives, thereby reshaping manufacturers' profit structures and financing choices.From a managerial and policy standpoint, our analysis suggests several actionable pathways: raising consumer environmental awareness, developing targeted green financing instruments such as green bonds, and fostering innovation in remanufacturing and eco-design. These measures can collectively enhance the sustainability and economic viability of closed-loop supply chains. In essence, coordinating demand-side policy interventions with appropriately structured financing mechanisms offers a promising avenue for achieving dual economic and environmental objectives in green supply chain operations.

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